where $N_{Nu_x} = hx/k$, the Nusselt number based on the distance from the slot, $N_{Re_x} = xU_f\rho/\mu$, the Reynolds number based on the distance from the slot and the belt speed, and $N_{Pr} = C_p\,\mu/k$, which we assumed to be a constant for our sets of experiments, is shown in Figure 2 as a solid line, along with our data.

We calculated the belt temperature drop from the slot to the return slot from an overall heat balance and from the method suggested in the appendix of Griffith's paper and found it to be on the order of 2 to 5°F. for our set of runs, supporting our assumption of isothermal belt temperature.

We found that in the region of ½ in. from the slot, the experimental heat transfer coefficients were 10 to 25% lower than those predicted by the Erickson theory. We attribute this to the presence of a vertical wall and the corresponding movement of air down the wall and from the edges of the belt, replacing that which has been dragged along by the belt. The analysis of this three-dimensional boundary layer in this region was not attempted. At a distance approaching one ft. from the slot, the experimental values were 10 to 60% higher than those predicted. We attribute this to natural convection heat transfer. Increasing belt speed caused an expected decrease in the natural convection effect and an expected increase in the wall effect.

It appears that, within experimental error discussed above, we are in good agreement with the theoretical prediction with the theory of Erickson et al.

Further studies using continuously extruded sheet and a laser grating interferometer are in progress.

ACKNOWLEDGMENT

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NOTATION

C_p = specific heat, air, B.t.u./lb. °F.
h = forced convection heat transfer coefficient, B.t.u./sq.ft. °F.
k = thermal conductivity, air, B.t.u./hr. sq.ft. °F./ft.
U_f = belt velocity, ft./sec.
x = distance along belt, from slot, in.
μ = viscosity, air, centipoise
ρ = density, air, lb./cu.ft.
N_{Nu_x} = Nusselt number, based on belt distance from slot, dimensionless
N_{Re_x} = Reynolds number, based on belt distance from slot and belt velocity, dimensionless
N_{Pr} = Prandtl number, for air, dimensionless

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Use of Time Delays in Packed Gas Absorption Column Simulation

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There have been relatively few studies of the dynamic behavior of packed absorption or distillation columns. This is probably because such continuous systems involve more than one space variable and have a complex geometry. For these reasons it is difficult to formulate a meaningful theoretical transfer function for packed columns. However, two very different approaches to the analysis of such distributed parameter systems are already evident (8). These are (*a*) successively more rigorous partial differential equation models that attempt to explain the dynamic behavior by a detailed analysis of the mechanisms involved, or (*b*) relatively simple, highly approximate ordinary differential equation models that are suitable for control purposes. The second approach has been applied to parallel simulation and other feedforward control schemes that require a simple, approximate process model (8). This paper, which is concerned with the second approach, shows that the inclusion of time delay elements in a mixing-cell model gives a simpler analog computer model than previously reported.

If we neglect radial concentration changes, the two-phase packed absorption column can be represented by partial differential equations involving only time and one space variable as independent variables (3). By applying finite difference techniques, these partial differential mass balances for each phase can be reduced to a set of ordinary difference equations. Since these are similar to the mass balances of staged process equipment, it is logical to use a staged mixing-cell model to represent the set of ordinary difference equations. Gray (3, 4) used a staged CSTR model consisting of mixing cells in series for each phase. The component mass balances used by Gray were

$$x_{n+1}\,L - x_nL + RaH = Hh_L\frac{dx_n}{dt} \qquad (1)$$

for the liquid-phase cells and

$$y_{n+1}\,G - y_nG - RaH = Hh_G\frac{dy_n}{dt} \qquad (2)$$

for the vapor-phase cells.

Since the packed column is a distributed parameter system, the inclusion of time delays (or transportation lags) in the mixing cell model should help account for the relatively stagnant fluid regions found in packed columns and for the finite transport time between stages. The time delay does not affect the steady state behavior of the column. Others have reported on the use of time delays in dynamic models of distributed parameter systems (*1, 6*).

Since the time delay does not affect the steady state, the minimum number of mixing stages needed in the model can be determined from the steady state response. The approximation to the actual steady state response should improve with an increasing number of stages because the model approximates a continuous system with a finite number of stages. In the limit of an infinite number of differential stages, the steady state model equations are the same as the equations normally used to calculate overall mass transfer coefficients from experimental column data. Therefore, if reported mass transfer coefficients are used in the mixing cell model, the reported concentration levels should be approached asymptotically with an increasing number of stages. Since each additional stage requires additional simulation hardware, there will be a point at which the improvement is so small that additional stages are not justified. The steady state results of several investigations (*7, 11*) were used in mixing cell models without time delays and it was found that the use of only two stages for each phase gave steady state computer results that were within 5% of the experimental results reported. Many columns would probably require more than two stages of perfect mixers, but the above agreement suggests that a two-stage mixing cell model without time delays is a reasonable approximation of the steady state.

A two-stage mixing cell model with time delays was then developed to simulate the dynamic behavior of the packed column. This model is illustrated schematically in Figure 1. In order to find values for the time constants and delay times, the approach of Hoerner (*5*) is used to separate the residence times Zh_L/L and Zh_G/G into time constants and delay times. For the two-stage model, the residence time is $2\tau + \tau_D$ if both of the time constants are assumed to be identical.

The operating and static holdup data of Shulman (*9*) provide a convenient means of estimating the liquid parameters. Shulman separated the liquid holdup into an operating holdup h_0 and a static holdup h_S. If it is assumed that the operating holdup is in the mixing cells, and the static holdup is in the time delay, then for the two-stage, delay time model the liquid residence time can be separated to give

$$\frac{Zh_L}{L} = 2\left(\frac{Zh_{LO}}{2L}\right) + \frac{Zh_{LS}}{L} \qquad (3)$$

There is some physical basis for this approach. Various investigators (*2*) have noted that there are poorly mixed regions in the liquid phase of a packed bed. These regions appear to contribute little to the mass transfer. This suggests that the fraction of the residence time to be attributed to the delay time should be partially dependent on the static holdup.

The total vapor holdup h_G can be calculated from the void fraction and the liquid holdup. Since there is no corresponding way to separate the vapor holdup, it is assumed that the vapor holdup can be separated in the same ratio as the liquid holdup so that

$$h_{GO} = h_G\left(\frac{h_{LO}}{h_L}\right) \qquad (4)$$

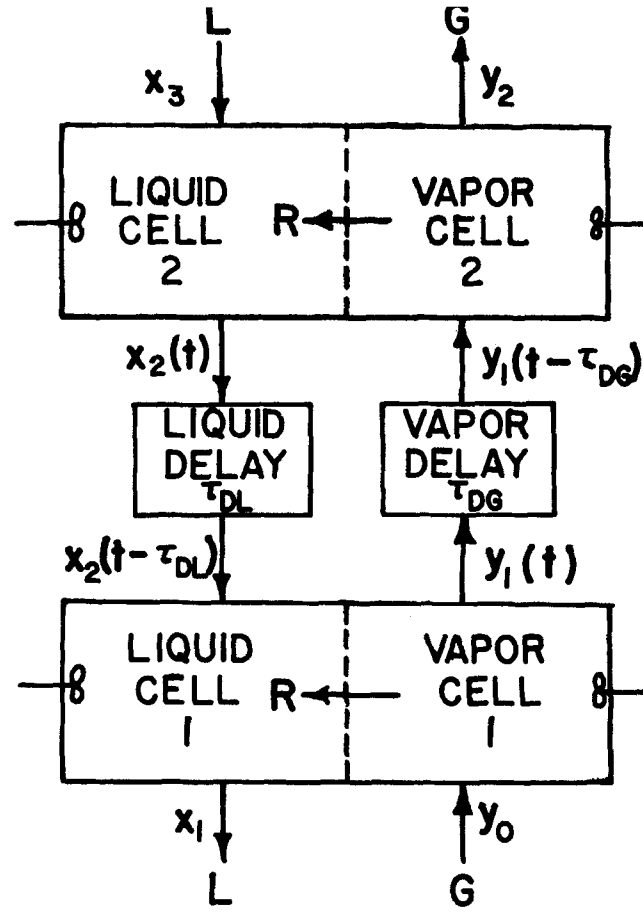

Fig. 1. Two stage mixing cell-delay model.

and

$$h_{GS} = h_G\left(\frac{h_{LS}}{h_L}\right) \qquad (5)$$

The analog program was then developed using the following approximations:

1) liquid film controlled mass transfer $Ra = K_x a(x^* - x)$
2) linear phase equilibrium relation $y = m\,x^*$
3) Stubbs-Single (*10*) eighth order time delay approximation

The first two assumptions were used by Gray, with whom comparison is made, and the last is a common analog approximation. The resulting analog model equations are

$$\frac{dx_2}{dt} = \frac{L}{Hh_{LO}}x_3 - \left(\frac{L}{Hh_{LO}} + \frac{K_x a}{h_{LO}}\right)x_2 + \frac{K_x a}{h_{LO}m}y_2 \qquad (6)$$

Table 1. Typical Parameters

System: Air-carbon dioxide-water (Gray)
Column: 6 in. diameter by 5.12 feet packed length (Gray)
Liquid flow: 55 lb. moles/hr.-ft.^{2} (Gray)
Vapor flow: 0.87 lb. moles/hr.-ft.^{2} (Gray)
Inlet vapor concentration: $y_0 = 0.100$ (Gray)
Mass transfer coefficient: $K_x a = 90$ lb. moles/hr.-ft.^{2} (Gray)
Equilibrium slope $m = y/x^* = 1{,}565$ (Gray)
Total liquid holdup: 0.19 lb. moles/cu. ft. column (Shulman)
Operating liquid holdup: 0.0935 lb. moles/cu. ft. column (Shulman)
Static liquid holdup: 0.097 lb. moles/cu. ft. column (Shulman)
Total vapor holdup: 0.00139 lb. moles/cu. ft. column

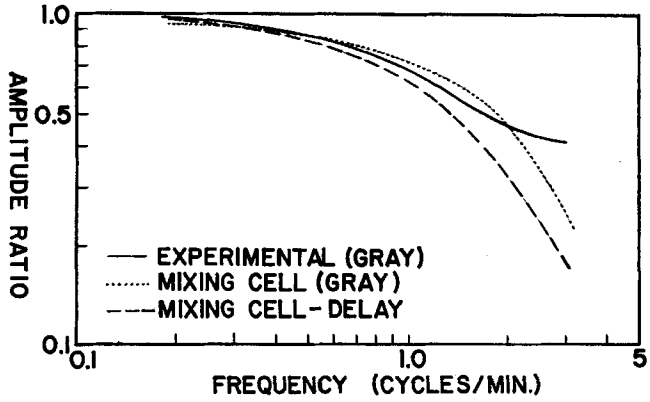

Fig. 2. Amplitude ratio comparison (Run 3, Gray).

$$\frac{dx_1}{dt} = \frac{L}{Hh_{LO}} x_{2D} - \left(\frac{L}{Hh_{LO}} + \frac{K_x a}{h_{LO}} \right) x_1 + \frac{K_x a}{h_{LO} m} y_1 \quad (7)$$

$$\frac{dy_2}{dt} = \frac{G}{Hh_{GO}} y_{1D} - \left(\frac{G}{Hh_{GO}} + \frac{K_x a}{h_{GO} m} \right) y_2 + \frac{K_x a}{h_{GO}} x_2 \quad (8)$$

$$\frac{dy_1}{dt} = \frac{G}{Hh_{GO}} y_0 - \left(\frac{G}{Hh_{GO}} + \frac{K_x a}{h_{GO} m} \right) y_1 + \frac{K_x a}{h_{GO}} x_1 \quad (9)$$

$$y_{1D}(t) = y_1(t - \tau_{DG}) \quad (10)$$

$$x_{2D}(t) = x_2(t - \tau_{DL}) \quad (11)$$

where $\tau_{DG} = Zh_{GS}/G$ and $\tau_{DL} = Zh_{LS}/L$.

The frequency response of the computer model was measured by forcing the inlet gas phase concentration y_0 sinusoidally. Typical experimental column parameters of Gray (3) and Shulman (9) that were used are given in Table 1.

The frequency response of the two-stage mixing cell delay model was compared with that of Gray's (3) sixty-stage mixing cell model and with Gray's experimental response. Gray's experimental data were severely limited at high frequencies and the linearity assumption used to eliminate end effects reduced the accuracy considerably. However, these are the best comparative data available. The two-stage mixing cell delay model represents the experimental response about as well as the sixty-stage mixing cell model. Typical results are shown in Figures 2 and 3, where comparison is made with Gray's Run 3.

The use of only two mixing stages with time delays requires about 15% of the analog hardware required by the sixty-stage mixing model. Therefore, the inclusion of time delays represents a significant simplification in packed absorption column simulation. The model is suitable for adaptive feedforward control schemes in which initial model parameters could be determined as above and adaptive parameter changes made by a comparison of model and column response. Even in cases where steady state data indicate the need for more stages, the staged mixing cell delay model probably represents a suitable model for most packed absorption column control schemes that require an approximate process model.

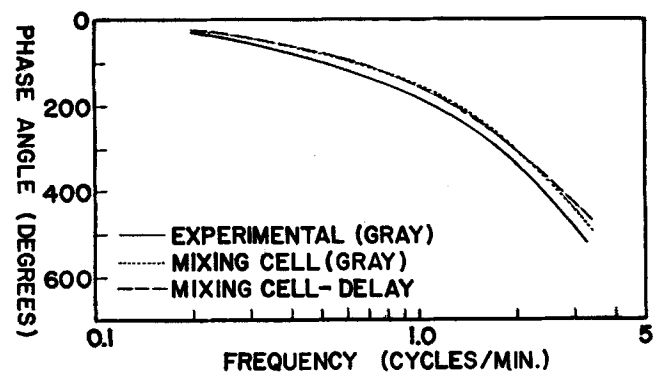

Fig. 3. Phase angle comparison (Run 3, Gray).

NOTATION

- a = mass transfer area, sq.ft. interfacial area/cu.ft. column
- G = gas phase molar velocity, lb. mole/hr.-sq.ft. column cross-sectional area
- H = Z/N packing height equivalent, ft.
- h_G = total gas phase holdup, lb. mole/cu.ft. column
- h_{GO} = gas phase operating holdup, lb. moles liquid/cu.ft. column
- h_{GS} = gas phase static holdup, lb. moles gas/cu.ft. column
- h_L = total liquid phase holdup, lb. mole/cu.ft. column
- h_{LO} = liquid phase operating holdup, lb. moles liquid/cu.ft. column
- h_{LS} = liquid phase static holdup, lb. moles liquid/cu.ft. column
- K_x = mass transfer coefficient, lb. mole/hr.-ft.^{2}-mole fraction
- L = liquid phase molar velocity, lb. mole/hr.-sq.ft. column
- m = slope of equilibrium line, mole fraction vapor/mole fraction liquid
- N = number of mixing cell stages
- R = mass transfer rate, lb. moles/hr.-sq.ft. interfacial area
- t = time
- x = mole fraction solute in liquid phase
- x^* = liquid mole fraction in equilibrium with bulk gas phase mole fraction at that point
- y = mole fraction solute in gas phase
- Z = packing length, ft. of column

Greek Letters

- τ_{DG} = Zh_{GS}/G, delay time for gas phase, hr.
- τ_{DL} = Zh_{LS}/L, delay time for liquid phase, hr.
- τ_G = Hh_{GO}/G, gas phase time constant, hr.
- τ_L = Hh_{LO}/L, liquid phase time constant, hr.

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